INTERNATIONAL JOURNAL OF R&D IN ENGINEERING SCIENCE AND MANAGEMENT

Research Paper

Estimation of Extreme Wind Speed Using L-Moments of Probability Distributions

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ABSTRACT

Estimation of Extreme Wind Speed (EWS) is one of the important parameter for planning, design and management of civil structures such as bridges, wind turbines and radio masts etc. This can be achieved through Extreme Value Analysis (EVA) of wind speed data adopting five probability distributions. Method of L-Moments (LMOM) is applied for determination of parameters of the distributions for estimation of EWS. Goodness-of-Fit tests such as Chi-Square and Kolmogorov-Smirnov are applied for checking the adequacy of fitting of distributions to the recorded data. A diagnostic test of D-index is used for the selection of a suitable distribution for EVA of wind speed. The paper presents the Gumbel Extreme Value is better suited amongst five distributions studied for estimation of EWS for Delhi and Extreme Value Type-1 distribution for Kanyakumari.

Keywords: Chi-Square, D-index, Kolmogorov-Smirnov, Probability distribution, Wind speed

1. Introduction

High wind speeds pose a threat to the integrity of structures, particularly those at exposed sites such as bridges, wind turbines and radio masts. In any design project for large structures, safety considerations must be balanced against the additional cost of 'over-design'. Accurate estimation of the occurrence of Extreme Wind Speed (EWS) is an important factor in achieving the correct balance. Such estimate is expressed in terms of the quantile value (x_T), viz., the maximum wind speed which is exceeded, on average, once every T-year (yr) [1]. This can be achieved through Extreme Value Analysis (EVA) of wind speed data adopting five probability distributions.

Number of probability distributions like Exponential (EXP), Generalized Extreme Value (GEV), Generalized Pareto (GPA), Gumbel (EV1) and Normal (NOR) are widely used for estimation of extreme events such as rainfall, flood, low-flow and wind speed. Generally, method of moments (MOM) is used for determination of parameters of the distributions. But, the MOM is not giving satisfactory results though the method exists for a longer period [2-5]. It is sometimes difficult to assess exactly (i) what information about the shape of a distribution is conveyed by its moments of third and higher order; (ii) the numerical values of sample moments particularly when the sample is small, can be very different from those of the probability distribution from which the sample was drawn; and (iii) the estimated parameters of distributions fitted by the MOM are often less accurate than those obtained by other estimation procedures such as Maximum Likelihood Method (MLM), method of least squares and probability weighted moments [6]. To overcome this, the alternative approach, namely L-Moments (LMOM) is discussed and also used for EVA of wind speed.

In the recent past, number of studies has been carried out by different researchers on adoption of probability distributions for EVA of wind speed. Palutikof et al. [7] expressed that the GEV distribution is better suited for Sumburgh (Shetland) amongst GEV and GPA distribution studied for frequency analysis of wind speed. Pandey et al. [8] applied GEV and Gamma distributions for estimation of EWS for Helena, Boise and Duluth stations in United States of America. Shabri [9] carried out the study on regional frequency analysis of annual maximum rainfall for Selangor and Kuala Lumpur, Malaysia adopting nine probability distributions (using LMOM). Kunz et al. [10] compared the Gamma and GPA distributions for estimation of EWS and concluded that GPA provides better estimates than Gamma. El-Shanshoury and Ramadan [11] adopted EV1 distribution (using LMOM and MOM) to estimate EWS for Dabaa area in the north-western coast of Egypt. From these studies, it is observed that there is an agreement in applying GEV and EV1 distributions for EVA of wind speed though number of distributions is available for modelling wind speed data. The objective of the paper is to compare the EWSs obtained from five probability distributions (using LMOM) and to identify the best suitable distribution for EVA of wind speed for Delhi and Kanyakumari. Goodness-of-Fit (GoF) tests such as Chi-Square (χ^2) and Kolmogorov-Smirnov (KS) are applied for checking the adequacy of fitting of distributions to the recorded wind speed data. A diagnostic test of D-index is used for the selection of a suitable probability distribution for EVA of wind speed. The procedures involved in determining the parameters of the distribution (using LMOM), estimation of EWSs for different return periods, computation of GoF tests statistic and D-index are briefly described in the following sections.

2. Methodology

LMOM are analogous to the conventional moments but can be estimated linear combination of order statistics, i.e., by L-statistics. LMOM are less subject to bias in estimation and approximate their asymptotic normal distribution more closely in finite samples. Parameter estimates obtained from LMOM are generally more accurate than MLM estimates for small sample.

2.1 Theoretical Description of LMOM

Method of LMOM is a modification of the probability weighted moments method explored by Hosking and Wallis [12]. Parameters of the distribution are estimated by equating the sample LMOM (l_r) with the distribution of LMOM (b_r). In practice, LMOM must be estimated from a finite sample. Let $x_{1n} \le x_{2n\le \ldots \le} x_{nn}$ be the ordered sample of size n. The sample LMOM is given by:

$$l_{r+1} = \sum_{k=0}^{r} \frac{(-1)^{r-k}(r+k)!}{(k!)^2(r-k)!} b_k \quad (1)$$

where,

$$b_k = n^{-1} \sum_{i=k+1}^{n} \frac{(i-1)(i-2).....(i-k)}{(n-1)(n-2).....(n-k)} x_{1n}$$

The sample LMOM ratios are defined as

$$t_2 = l_2 / l_1 \text{ and } t_r = l_r / l_2 \text{, r=3,4,5,.....} \quad (2)$$

The first two sample LMOM is expressed by

$$l_1 = b_0 \text{ and } l_2 = 2b_1 - b_0 \quad (3)$$

Table 1 gives the LMOM and parameters of five probability distributions used for EVA of wind speed [13].

2.2 Goodness-of-Fit Tests

For checking the adequacy of fitting of probability distributions to the recorded data, χ^2 and KS tests are applied. Theoretical description of χ^2 statistic is as follows:

$$\chi^2 = \sum_{j=1}^{NC} \frac{\left(O_j(x) - E_j(x)\right)^2}{E_j(x)} \quad (4)$$

The rejection region of χ^2 statistic at the desired significance level (η) is $\chi^2_C \ge \chi^2_{1-\eta,NC-m-1}$. Here, 'm' denotes the number of parameters of the distribution.

The KS statistic is defined by

$$KS = \overset{n}{\underset{i=1}{Max}}\left(F_e(x_i) - F_D(x_i)\right) \tag{5}$$

where, $F_e(x_i) = (i-0.44)/(n+0.12)$ and 'i' is the rank assigned to the sample values arranged in ascending order. The critical value (KS_C) of KS statistic for different sample size (n) at 5 percent significance level is computed from $KS_C = 1.36/\sqrt{n}$ (for n>35).

Test criteria: If the computed values of GoF tests statistic given by the distribution are lesser than that of the critical values at the desired significance level, then the distribution is considered to be acceptable for EVA of wind speed [14].

Table 1 - LMOM and parameters of probability distributions

Distribution	Quantile function	LMOM	Parameters
Exponential (EXP)	$x_T = \psi - \bar{x}\log(1-F)$	$l_1 = \psi + \bar{x}$; $l_2 = \bar{x}/2$	ψ (known); $\bar{x} = l_1$
Generalised Extreme Value (GEV)	$x_T = \xi + \alpha(1-(-\log F)^k)/k$	$l_1 = \xi + \alpha(1-\Gamma(1+k))/k$; $l_2 = \alpha(1-2^{-k})\Gamma(1+k)/k$	$z = (2/(3+t_3) - (\log 2/\log 3)$; $k = 7.8590\ z + 2.9554\ z^2$; $\alpha = l_2 k/(1-2^{-k})\Gamma(1+k)$; $\xi = l_1 + (\alpha(\Gamma(1+k)-1)/k)$
Generalised Pareto (GPA)	$x_T = \xi + \alpha(1-(1-F)^k)/k$	$l_1 = \xi + (\alpha/(1+k))$; $l_2 = \alpha/(1+k)(2+k)$	$\xi = l_1 + l_2(k+2)$; $k = (4/(t_3+1)) - 3$; $\alpha = (1+k)(2+k)l_2$
Gumbel (EV1)	$x_T = \xi - \alpha\log(-\log F)$	$l_1 = \xi + \gamma\alpha$; $l_2 = \alpha\log 2$	$\alpha = l_2/\log 2$; $\xi = l_1 - \gamma\alpha$
Normal (NOR)	$x_T = \bar{x} + \sigma\phi^{-1}(F)$	$l_1 = \bar{x}$; $l_2 = (1/\sqrt{\pi})\sigma$	$\bar{x} = l_1$; $\sigma = l_2\sqrt{\pi}$
Note: $\phi^{-1} = z_P = (P^{0.135} - (1-P)^{0.135})/0.1975$ and P is the probability of exceedance (F(x)=P(X≤x)=1/T).			

2.3 Diagnostic Test

The selection of a suitable probability distribution for EVA of wind speed is performed through D-index though GoF tests give sufficient information on fitting of probability distributions to the recorded data. The D-index is defined by:

$$\text{D-index} = \left(1/\bar{x}\right)\sum_{i=1}^{6}\left|x_i - x_i^*\right| \tag{6}$$

Here, x_i 's (i=1 to 6) are the first six highest sample values in the series. The distribution having the least D-index is adjudged as better suited distribution for EVA of wind speed [15].

3. Application

An attempt has been made to estimate the EWS for different return periods for Delhi and Kanyakumari adopting five probability distributions (using LMOM). Hourly wind speed data recorded at Delhi for the period 1969-2007 and Kanyakumari for the period 1970-2008 are used. The series of annual hourly maximum wind speed is extracted from the hourly data and used for EVA. Table 2 gives the descriptive statistics of annual hourly maximum wind speed for the regions under study.

Table 2 - Descriptive statistics of annual hourly maximum wind speed

Region	Summary statistics			
	Mean (km/hr)	Variance (km/hr)^2	Skewness	Kurtosis
Delhi	66.1	261.1	0.047	-1.709
Kanyakumari	42.3	123.0	2.219	6.848

4. Results and Discussions

By adopting the procedures of LMOM of the probability distributions, as detailed above, computer program through R-package was developed and used for EVA of wind speed. The program computes the LMOM and parameters of the distributions, GoF tests statistic and D-index values for Delhi and Kanyakumari.

4.1 Estimation of EWSs using probability distributions

The parameters obtained from LMOM were used for estimation of EWS for Delhi and Kanyakumari through quantile functions of the respective probability distributions and given in Tables 3 and 4. The estimated EWSs were further used to develop the frequency curves and presented in Figures 1 and 2.

Table 3 - Estimated EWS using probability distributions for Delhi

Return period (yr)	Extreme Wind Speed (km/ hr)				
	EXP	GEV	GPA	EV1	NOR
2	60.5	66.3	66.5	63.4	66.1
5	77.2	80.2	82.6	78.3	79.7
10	89.9	87.1	87.7	88.1	86.8
20	102.5	92.3	90.1	97.6	92.7
50	119.2	97.5	91.4	109.9	99.3
100	131.9	100.5	91.9	119.0	103.7
200	144.5	103.0	92.1	128.2	107.8
500	161.2	105.5	92.2	140.3	112.6
1000	173.9	106.9	92.2	149.4	116.1

Table 4 - Estimated EWS using probability distributions for Kanyakumari

Return period (yr)	Extreme Wind Speed (km/ hr)				
	EXP	GEV	GPA	EV1	NOR
2	38.9	39.3	38.9	40.6	42.3
5	48.9	47.7	48.9	49.5	50.4
10	56.5	54.8	56.5	55.4	54.7
20	64.1	62.9	64.1	61.1	58.2
50	74.1	75.6	74.1	68.4	62.1
100	81.6	87.2	81.7	73.9	64.8
200	89.2	100.9	89.4	79.4	67.2
500	99.2	122.9	99.5	86.7	70.1
1000	106.8	143.1	107.1	92.1	72.2

From Table 3, it may be noted that the estimated EWSs (using EXP) are consistently higher than the corresponding values of other four distributions for return periods of 10-yr and above. From Table 4, it may be noted that there is no appreciable between the estimated values given by EXP and GPA distributions. For Kanyakumari, GEV distribution gave higher estimates for different return periods of 50-yr and above when compared to the corresponding values of other four distributions.

4.2 Analysis based on GoF tests

For the present study, the degree of freedom (NC-m-1) was considered as two for 3-parameter distributions (GEV and GPA) and three for 2-parameter distributions (EXP, EV1 and NOR) while computing the χ^2 statistic values. GoF tests statistic values were computed from Eqs. (4) and (5), and given in Table 5.

Table 5: Computed values of GoF tests statistic

Probability distribution	Computed values of GoF tests statistic for			
	Delhi		Kanyakumari	
	χ^2	KS	χ^2	KS
EXP	12.923	0.194	6.744	0.123
GEV	11.897	0.199	3.872	0.088
GPA	14.974	0.172	6.744	0.207
EV1	15.231	0.200	7.154	0.127
NOR	14.462	0.204	10.846	0.192

From Table 5, it may be noted that the computed values of χ^2 statistic are greater than the theoretical values ($\chi^2_{0.05,2}$ =5.99 and $\chi^2_{0.05,3}$ =7.81) at 5 percent significance level, and at this level, all five distributions are not acceptable for EVA of wind speed for Delhi. For Kanyakumari, the distributions other than GPA and NOR are found to be acceptable for EVA of wind speed, which was confirmed by χ^2 test results. Also, from Table 5, it may be noted that the computed values of KS statistic are lesser than the theoretical values of 0.218 at 5 percent significance level, and at this level, the five distributions are acceptable for EVA of wind speed for the regions under study.

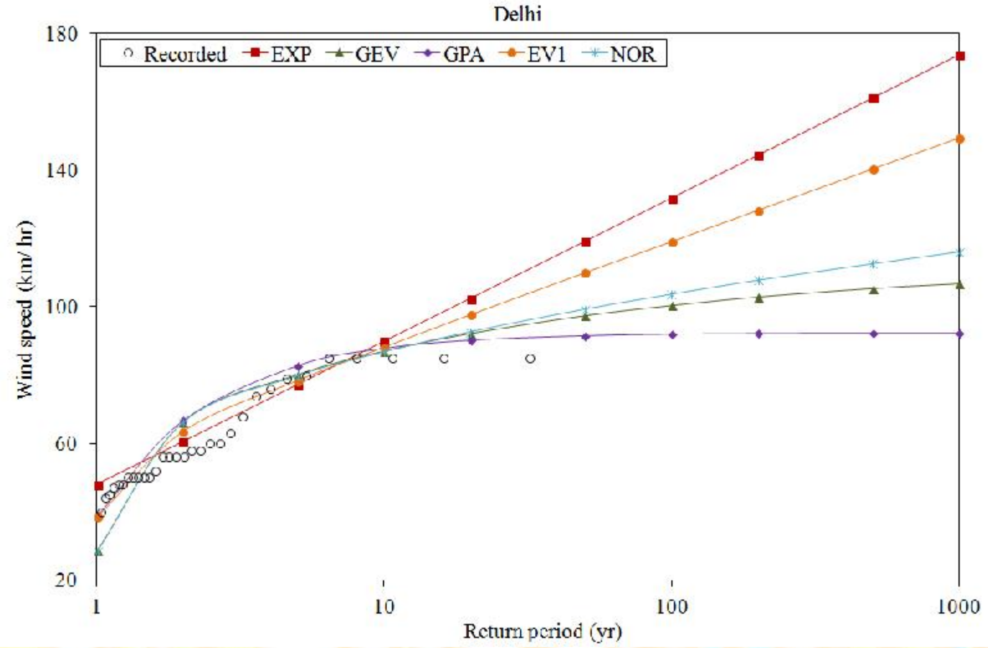

Fig. 1 - Plots of recorded and estimated extreme wind speeds for different return periods by five probability distributions (using LMOM) for Delhi

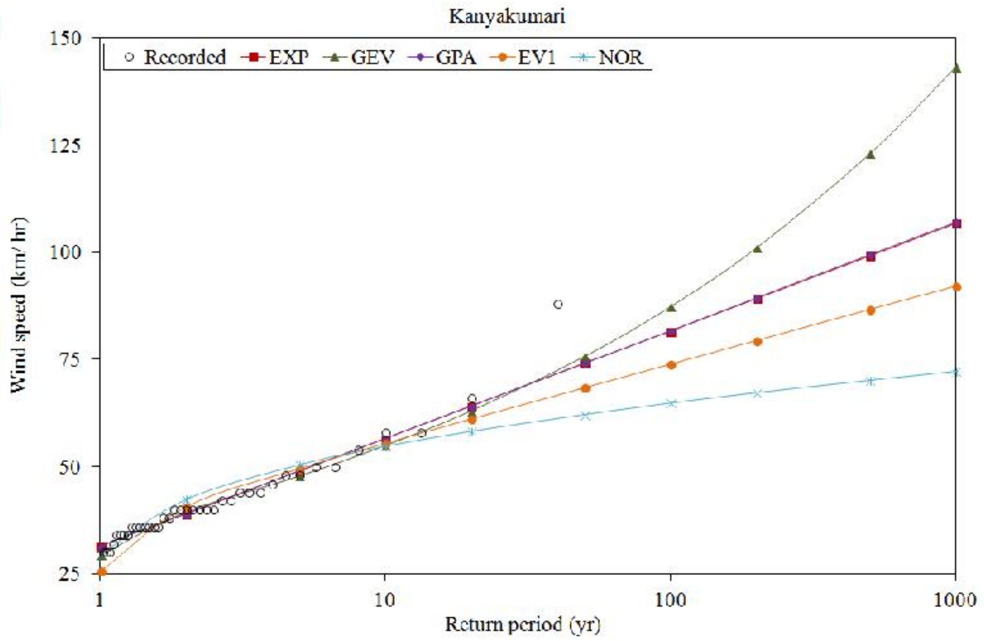

Fig. 2 - Plots of recorded and estimated extreme wind speeds for different return periods by five probability distributions (using LMOM) for Kanyakumari

4.3 Analysis based on diagnostic test

In addition to GoF tests, the D-index values of the probability distributions were also computed by Eq. (6) and given in Table 6.

Table 6: D-index values of probability distributions for Delhi and Kanyakumari

Region	Indices of D-index				
	EXP	GEV	GPA	EV1	NOR
Delhi	4.101	1.377	0.618	2.979	1.648
Kanyakumari	2.907	3.354	2.917	2.217	1.467

From Table 6, it may be noted that the indices of D-index of 0.618 (using GPA) for Delhi and 1.467 (using NOR) for Kanyakumari are minimum when compared to the corresponding values of other distributions. But, both GPA and NOR distributions were rejected by χ^2 test though the distributions have minimum D-index. Also, from Table 6, it may be noted that the indices of D-index of 1.377 (using GEV) and 2.217 (using EV1) are the second minimum next to the first one given by GPA and NOR for Delhi and Kanyakumari respectively. Based on KS and diagnostic test results, GEV distribution is found to be a good choice for estimation of EWS for Delhi whereas EV1 for Kanyakumari.

5. Conclusions

The paper presented a computer aided procedure for determination of parameters of five probability distributions (using LMOM) for estimation of EWS for Delhi and Kanyakumari. The study showed that the selection of a suitable distribution was evaluated by GoF (using χ^2 and KS) and diagnostic (using D-index) tests. The χ^2 test results indicated that both GPA and NOR distributions were not found to be acceptable for fitting the wind speed data though the D-index values of the distributions were found to be minimum when compared to the other distributions for Delhi and Kanyakumari respectively. On the basis of KS and diagnostic test results, the GEV is found to be better suited distribution for estimation of EWS for Delhi whereas EV1 for Kanyakumari. The study suggested that the 1000-yr return period EWS of 106.9 km/ hr (using GEV) for Delhi and 92.1 km/ hr (using EV1) obtained from EVA of wind speed could be considered while designing the proposed civil structures in the regions. The plots of the recorded and estimated EWSs by five probability distributions (using LMOM) were developed and presented in the paper.

Acknowledgements

The author is grateful to the Director, Central Water and Power Research Station, Pune for providing the research facilities to carry out the study. The author is thankful to the India Meteorological Department, Pune and M/s Nuclear Power Corporation of India Limited, Mumbai for supply of wind speed data.

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Abbreviations

CDF	:	Cumulative Distribution Function
EVA	:	Extreme Value Analysis
EWS	:	Extreme Wind Speed
EV1	:	Extreme Value Type-1
EXP	:	Exponential
$E_j(x)$	:	Expected frequency value of j^{th} class
F(x) (or F)	:	CDF of x
$F_D(x_i)$	:	Computed CDF of x_i
$F_e(x_i)$	:	Empirical CDF of x_i
GEV	:	Generalized Extreme Value
GPA	:	Generalized Pareto
LMOM	:	L-moments
MLM	:	Maximum Likelihood Method
NC	:	Number of frequency classes
NOR	:	Normal
$O_j(x)$	:	Observed frequency value of j^{th} class
l_{r+1}	:	$r+1^{th}$ sample moment
b_k	:	Unbiased estimator of β_k
t_3	:	L-skewness
t_4	:	L-kurtosis
γ	:	Euler's constant
ϕ^{-1}	:	Inverse of the standard normal distribution function
χ^2	:	Chi-square
ξ	:	Location parameter
α	:	Scale parameter
k	:	Shape parameter
σ	:	Standard deviation
KS	:	Kolmogorov-Smirnov
$\overline{x}$	:	Mean value of the recorded data
x_i	:	Recorded value of the i^{th} sample
x_i^*	:	Estimated value by the probability distribution